OUTLINE OF INVESTIGATION FOR ASYMPTOTES.

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I. Methods.

A. By Inspection.

1. If when $x \doteq 0$, $y \doteq \infty$, $x=0$ is an asymptote.
2. " " $x \doteq 0$, $y \doteq a$, or $\sqrt{-a}$, $x=0$ is not an asymptote.
3. " " $x \doteq \pm a$, $y \doteq \pm\infty$, $x=\pm a$ is an asymptote.
4. " " $x \doteq \pm a$, $y \doteq \pm a$, or $\pm\sqrt{-a}$, $x=\pm a$ is not an asymptote.
5. Treat y in some manner as x.

Note 1. To be universally true, the equation must be solved for either y or x.

Note 2. The finite quantity is seen in the curve.

B. By direct Investigation.

1. Through Intercepts.

(a). If X and Y, either or both, are finite, there is an asymptote.

(b). If X and Y are both infinite, no asymptotes.

(c). To find the equation of the asymptote, substitute the intercepts of X and Y for a and b in $\dfrac{x}{a}+\dfrac{y}{b}=1$.

Note. Under (a), if need be, find the limit of $\dfrac{y}{x}$.

2. By writing for y, $kx+r$, in the curve, expanding, arranging according to the descending powers of x, writing the coefficients of the two highest powers of x equal to 0, from which find the values of k and r, which values substituted in last two terms of arranged equation just found give the equation of an asymptote.

3. By solving the equation for y and developing by Maclaurin's formula, etc.

II. An example of application and illustration. I shall take the cissoid (of Diocles), because I have never seen it worked, either by text or student, except by the method of inspection. Of course by inspection, by 3rd under A above it is instantly seen that $x=2a$ is an asymptote. I have had students declare that it could not be solved by direct investigation. It comes directly under (a) of B above. First, as in Analytic Geometry, investigate for limits of the curve. We discover it is limited by $2a$ to the right. From $y^2=\dfrac{x^3}{2a-x}$, $\dfrac{dx}{dy}=\dfrac{2y(2a-x)}{3x^2+y^2}$ second in $X=x-y\dfrac{dx}{dy}$, sub. value of y and $\dfrac{dx}{dy}$, gives

$$x=\frac{ax^3}{3ax^2-x^3}=\frac{ax}{3a-x}=\frac{2a^2}{a}=2a, \text{ since } x=2a \text{ at limit.}$$

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fall on the earth at a', b', c'. The required curve, therefore, is a circle, as it should be; and the radius of this circle is the maximum range of the *fragments* and *balls*. Of course, if the atmospheric resistance and other complicating factors be not ignored, it is reasonable that the required curve should be somewhat *oval-shaped*—thus, to a certain extent, resembling the apparent disc of the rising sun or of the rising full moon.

20. **Proposed by SAMUEL HART WRIGHT, M. D., M. A., Ph. D., Penn Yan, Yates County, New York.**

When does the Dog-Star and the Sun *rise together* in latitude $\lambda=+42°\ 30'$, if the Right Ascension of the said star be $\alpha=6$ hrs., 40 min., 30 sec., and the declination $\delta=-16°\ 33'\ 56''$?

Solution by F. P. MATZ, M. Sc., Ph. D., Professor of Mathematics and Astronomy in New Windsor College, New Windsor, Maryland.

In order that "The Dog Star" and the Sun may rise together, their *hour-angles at the instant of rising* must be the same. According to *Chauvenet's Spherical and Practical Astronomy*, Vol. I., p. 218, Art. 153, we have the formula:

$$t=\pm[-\tan\lambda\tan\delta],=74°\ 10'\ 57''.77,=4h.\ 56m.\ 43.851\ sec.$$

Therefore, the next two *critical dates;* that is, the next two dates on which the *cosmical rising* of "The Dog Star" is possible in latitude $\lambda=+42°\ 30'$, must be May 2, 1895, and August 2, 1895. By the Right Ascension of "The Dog-Star," as given in the problem, we are led to consider August 2, 1895, as the required date.

NOTE—The next two critical dates with respect to the *cosmical setting* of "The Dog-Star," evidently, are May 9, 1895, and August 11, 1895; and of these dates, the required one is May 9, 1895.

Also solved by *Professor ZERR* and the *PROPOSER.*

NOTE—No one of our contributors has as yet been able to effect a full and satisfactory solution to problem 21.

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In $Y=y-x\dfrac{dy}{dx}$, sub. value of $\dfrac{dy}{dx}$, gives

$$Y=\frac{4ay^2-5xy^2-3x^3}{4ay-2xy}=\frac{4a-5x-\dfrac{3x^3}{y}}{\dfrac{4a}{y}-\dfrac{2x}{y}}=\frac{4a-5x}{0}=\frac{q}{0}=\infty,$$

since $y=\infty$ at limit.

Then by(c) under B, sub. values of X and Y, in $\frac{x}{a}+\frac{y}{b}=1$, gives

$\dfrac{x}{2a}+\dfrac{y}{\infty}=1.\ \therefore\ \dfrac{x}{2a}+0=1.\ \ \therefore\ x=2a$ which is the equation sought.